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# THE UNIFIED THEORY OF SHIP MOTIONS

by

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(Preliminary Copy of paper to be presented at the 13th Symposium on Naval Hydrodynamics Tokyo, Japan, 6-10 October 1980.)

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## 1. INTRODUCTION

Conventional ship hulls are slender in the geometrical sense, with small beam and draft compared to their length. This is convenient from the standpoint of hydrodynamic analysis, since slender-body approximations simplify the governing equation and boundary conditions.

Geometrical slenderness is sufficient to justify the classical slender-body theory of incompressible aerodynamics, but in ship hydrodynamics the wavelength represents an additional relevant length scale which must be considered in developing asymptotic theories of practical utility. This complication in slender-ship theory applies not only to unsteady motions in waves, but also to the analysis of steady-state wave resistance. The present paper is concerned only with the former problem, and is restricted to the solution of the radiation problem for forced heave and pitch motions in otherwise calm water. Work currently in progress by Sclavounos (1980) will extend this theory to the diffraction problem of incident waves, including the determination of the exciting forces and moments.

Substantial wave excitation in heave or pitch requires an incident wavelength greater than the ship length, typically by a factor of 1.5 or more. This implies a regime where, based on the beam and draft, a long-wavelength or low-frequency approximation is appropriate. Hydrostatic restoring forces and the Froude-Krylov exciting force are dominant, and the resulting theoretical description of ship motions is relatively simple. This is the leading-order result of "ordinary" slender-body theory.

For a ship proceeding with significant forward speed, the Doppler effect increases the frequency of encounter and shortens the radiated wavelength. Resonance occurs when this wavelength is comparable to the beam and draft, and therefore much less than the ship length. This is the applicable regime of strip theory, where three-dimensional interactions in the longitudinal direction are negligible.

The practical domain of ship motions in waves obviously embraces both of the above regimes, in the sense that the frequency of encounter may be low, especially for following seas, or high as in the case of a fast vessel in head seas. In the context of slender-body theory, it is desirable therefore to avoid restrictive assumptions concerning the wavelength or the frequency of encounter. That objective has led to the development of a "unified" slender body theory which embraces both long and short wavelengths in the sense defined above.

The theoretical framework for the unified theory of ship motions is developed in Newman (1978) and in more detail, for the special case of zero forward velocity, by Mays (1978). The latter work includes computations of the damping and added-mass coefficients for a floating spheroid, and the remarkable agreement of the latter with exact three-dimensional computations was an important motivation for extending the computations to ship-like forms, and to non-zero forward velocity. The present paper is intended to report on these efforts.

A brief review is given of the theoretical framework for the unified theory, in Section 2, and supplemented in Section 3 by the derivation of a simplified expression for the kernel function which governs longitudinal interactions along the ship's length. Numerical results for the added-mass and damping coefficients are presented in Section 4, to illustrate the practical utility of this theory in predictions of ship motions in waves. The unified theory also has been used to analyse the wave-energy absorption of elongated devices such as the Cockerell raft and Kaimei ship, and the results are described briefly in Section 5.

Before proceeding with the mathematical details of the unified theory, it may be useful to present a brief description which avoids so far as possible the use of mathematical arguments. The fundamental assumptions are that (1) the fluid motion is irrotational and incompressible, (2) the oscillatory motions of the ship and of the fluid are sufficiently small to linearize, and (3) the ship hull is geometrically slender.

For heave and pitch motions and, more generally, situations where the distribution of normal velocity on the hull surface is a slowly-varying function along the length, the flow is essentially two-dimensional in the near-field close to the hull. Changes in the x-direction are relatively small in this region, by comparison to changes in the transverse plane. Thus the flow in the near field is governed by the two-dimensional Laplace equation, and subject to the simplest linearized free-surface boundary condition which applies in two dimensions and is independent of forward velocity. These characteristics of the *inner* problem, and its solution, are similar to strip theory.

The *outer* problem which applies far from the hull surface is fully three-dimensional, with gradients in the longitudinal direction comparable to those in the transverse plane. The three-dimensional Laplace equation governs the solution, subject to the complete linearized free-surface boundary condition (where the forward speed is a significant parameter) and the radiation condition of outgoing waves at infinity.

Neither the inner nor the outer problem is unique, as described above, since nothing has been stated about their respective asymptotic behavior far away in the inner problem, and close to the ship in the outer problem. Following the method of matched asymptotic expansions, this nonuniqueness is resolved by requiring the two solutions to be compatible in a suitably defined *overlap* region.

In the special case of ordinary slender-body theory, the frequency is asymptotically small in the inner solution, and the "rigid" free-surface condition applies. For vertical motions of the ship's section there is a net source strength, and thus the inner solution is logarithmically singular at "infinity", in the overlap domain. As in the classical slender-body theory of aerodynamics, this determines the effective source strength of the outer solution. Conversely, the inner limit of the outer solution determines a nontrivial additive "constant" in the inner solution, which is a function of the longitudinal coordinate.

By comparison, in the high-frequency domain of strip theory, waves are present in the inner problem via the free-surface condition. Their outgoing radiated behavior at "infinity" can be matched directly to the inner limit of an appropriate three-dimensional source distribution, along the ship's axis. The inner free-surface condition does not admit an additive constant, and hence the inner solution is not affected by the matching process, justifying the strip-theory solution itself.

Proceeding without restriction of the frequency requires that the inner free-surface condition is preserved, as in the high-frequency case. However, the corresponding strip-theory solution with outgoing waves at "infinity" is not sufficiently general to match with the outer solution. Therefore a homogeneous solution is included in the inner problem, with standing waves at "infinity"; the coefficient of this homogeneous solution is determined from an integral equation similar to that which determines the additive constant in ordinary slender-body theory.

In summary, the unified solution is an extension of the ordinary slender-body theory and strip theory which apply respectively in the low- and high-frequency limits. The inner solution is similar to, but more general than that of strip theory. The two-dimensional damping and added-mass coefficients are the fundamental parameters of this inner solution but, with forward velocity, the complete solution of the kinematic hull boundary condition requires additional parameters to be evaluated. The integral equation associated with the matching requirement is an additional complication, but its solution is a relatively minor chore by comparison to the numerical procedures required in the strip theory. Thus, while the concept of the unified theory is a nontrivial extension of strip theory, the computational effort required to utilize this more general approach is not substantially greater. The numerical results which follow more than justify this additional effort.

## 2. THEORETICAL DERIVATION

We consider a ship which moves in the positive x-direction, with constant forward velocity $U$, while performing small harmonic oscillations of frequency $\omega$ in heave and pitch. These and other oscillatory quantities are expressed in complex form, with the time factor $e^{i\omega t}$ understood throughout. Both $U$ and $\omega$ are restricted to be >0. The analysis in this Section is abbreviated from Newman (1978), where more details are provided.

The principal task is to solve for the complex velocity potentials $\varphi_j$, due to heave (j=3) and pitch (j=5) motions of unit amplitude. With the assumptions stated in the Introduction, these potentials are governed by the three-dimensional Laplace equation

$$\varphi_{jxx} + \varphi_{jyy} + \varphi_{jzz} = 0 \qquad (1)$$

and, in the frame of reference moving with the steady forward velocity of the ship, by the linearized free-surface boundary condition

$$-\omega^2\varphi_j - 2i\omega U\varphi_{jx} + U^2\varphi_{jxx} + g\varphi_{jz} = 0, \quad \text{on } z = 0. \qquad (2)$$

Here $z = 0$ is the plane of the free surface and $z$ is positive upwards. Far from the ship the potentials $\varphi_j$ must satisfy a suitable radiation condition of outgoing waves and, for large depths, the condition of vanishing motion as $z \to -\infty$.

The potentials $\varphi_j$ are distinguished by their respective boundary conditions on the wetted surface of the ship hull. With the instantaneous position of this surface replaced by its steady-state mean $\bar{S}$, the appropriate boundary conditions are

$$\varphi_{3n} = i\omega n_3 + Um_3, \qquad (3)$$

$$\varphi_{5n} = -i\omega x n_3 - Uxm_3 + Un_3. \qquad (4)$$

Here the subscript $n$ denotes normal differentiation, with the unit normal vector pointing out of the fluid domain, $n_j$ is the component of this vector parallel to the $x_j$ axis, and $m_3$ is an auxiliary function defined in terms of the steady-state perturbation potential $U\bar{\phi}$ by the relation*

$$m_3 = -n_2\,\bar{\phi}_{yz} - n_3\,\bar{\phi}_{zz}, \text{ on } \bar{S}. \qquad (5)$$

Since $\bar{\phi}$ satisfies the rigid free-surface boundary condition in the inner region, $m_3$ is independent of $U$.

The boundary-value problem for $\varphi_j$ can be restated separately in the inner region, where the transverse radius $r = (y^2 + z^2)^{1/2}$ is small compared to the ship's length, and in the outer region where $r$ is large compared to the beam and draft. The radiation condition (and vanishing of the solution as $z \to -\infty$) are applicable only to the outer solution, and the boundary conditions (3) and (4) to the inner solution. The missing conditions in each case are replaced by the requirement of matching, in an overlap region where $r$ is large compared to the beam and draft but small compared to the length.

Gradients in the x-direction are neglected in solving the inner problem. The governing equation is

$$\varphi_{jyy} + \varphi_{jzz} = 0, \qquad (6)$$

subject to the free-surface boundary condition

$$-\omega^2\varphi_j + g\,\varphi_{jz} = 0, \qquad \text{on } z = 0 \qquad (7)$$

Equations (6) and (7) are applicable to the two-dimensional strip theory of ship motions. In view of the boundary conditions (3) and (4), particular solutions of the inner problem can be expressed in the form

$$\varphi_j^{(s)} = \phi_j + U\,\hat{\phi}_j, \qquad (8)$$

where the latter potentials satisfy (6), (7), and, on the hull profile, in planes $x$ = constant,

$$\phi_{3n} = i\omega n_3, \qquad (9)$$

$$\hat{\phi}_{3n} = m_3, \qquad (10)$$

$$\phi_5 = -x\phi_3, \qquad (11)$$

$$\hat{\phi}_5 = -x\hat{\phi}_3 - (i/\omega)\phi_3. \qquad (12)$$

*The subscripts j=1,2,3 correspond respectively to (x, y, z).

The potentials in (8) also satisfy the extraneous two-dimensional radiation condition. Thus we add to (8) a homogeneous solution of (6), (7), and of the boundary condition on the hull. This homogeneous solution can be obtained simply in the form $(\phi_3 + \bar{\phi}_3)$ where the overbar denotes the conjugate of the complex potential $\phi_3$. This homogeneous solution behaves like a two-dimensional standing wave at large distance from the hull, and can be regarded physically as the superposition of two diffraction solutions with symmetric incident waves acting upon the fixed hull profile.

In summary, the general solution of the inner problem takes the form

$$\varphi_j = \varphi_j^{(s)} + C_j(x)\ (\phi_3 + \bar{\phi}_3) \tag{13}$$

where the *interaction function* $C_j(x)$ is an arbitrary "constant" in the inner solution to be determined from matching.

The outer solution follows by considering the complete Laplace equation (1) and free-surface condition (2), but ignoring the hull boundary conditions. Assuming symmetry about the plane $y=0$, an appropriate solution follows from a longitudinal distribution of sources along the ship's length,

$$\varphi_j = \int_L q_j(\xi)\ G(x-\xi,\ y,\ z)\,d\xi. \tag{14}$$

Here $q_j(x)$ is the source strength, and $G$ denotes the potential of a "translating-pulsating" source situated on the x-axis at the point $x = \xi$. This potential is expressed generally in the form of a double Fourier integral over the free surface. Of particular utility in our analysis is the Fourier transform of $G$, with respect to $x$, which can be expressed as

$$G^*(y,\ z;\ k) = \int_{-\infty}^{\infty} G(x,\ y,\ z)\ e^{ikx}\,dx$$

$$= -\frac{1}{4\pi}\int_{-\infty}^{\infty} du\ \frac{\exp[z(k^2+u^2)^{1/2} + iyu]}{(k^2+u^2)^{1/2} - \nu}\ . \tag{15}$$

where*

$$\nu = (\omega + Uk)^2/g. \tag{16}$$

When $|k| < \nu$, there are two symmetric real poles in (15), and the appropriate contour of integration is deformed in their vicinity such that $\mathrm{Im}[u^2(\omega + Uk)] > 0$.

In order to match the inner approximation of (14) in the overlap region, an asymptotic approximation of (15) is required for small values of $(ky,\ kz)$. The desired result can be expressed in the form

$$G^*(y,z;k) = G_{2D} - \frac{1}{2\pi}\ (1 + Kz)f^*(k), \tag{17}$$

where $G_{2D} = G^*(y,z;0)$ is the two-dimensional source potential, which satifies (6) and (7), and

$$f^* = \ell n\ (2K/|k|) + \pi i - \frac{\cos^{-1}(\nu/|k|) - \pi}{(k^2/\nu^2 - 1)^{1/2}}\ ,$$

$$(k^2/\nu^2 > 1), \tag{18}$$

$$f^* = \ell n\ (2K/|k|) + \pi i - \frac{\cosh^{-1}(\nu/|k|) + \pi i\ \mathrm{sgn}\ (\omega + Uk)}{(1 - k^2/\nu^2)^{1/2}},$$

$$(k^2/\nu^2 < 1), \tag{19}$$

where $K = \omega^2/g$.

It remains to match the inner and outer solutions (13) and (14). This may be carried out in the Fourier domain, using the convolution theorem to transform (14), and the appropriate matching condition takes the form

$$\varphi_j^{(s)*} + [C_j(x)\ (\phi_3 + \bar{\phi}_3)]^* = q_j^*\ G^*. \tag{20}$$

Far from the hull in the inner domain, the two-dimensional potentials on the left side of (20) can be expressed in terms of the effective source strengths, in the form

$$\phi_j = \sigma_j\ G_{2D} \tag{21}$$

$$\dot{\phi}_j = \dot{\sigma}_j\ G_{2D} \tag{22}$$

Using (8), (17), and the fact that $\mathrm{Im}\ (G_{2D}) = \frac{1}{2}\ e^{Kz}\cos Ky$, and equating separately the factors of $G_{2D}$ in (20), it follows that

$$\sigma_j^* + U\dot{\sigma}_j^* + [C_j\ (\sigma_j + \bar{\sigma}_j)]^* = q_j^* \tag{23}$$

and

$$-i(C_j\bar{\sigma}_j)^* = -\frac{1}{2\pi}\ q_j^*\ f^*\ . \tag{24}$$

The error in the last equation is a factor $1 + O(K^2r^2)$.

*Equation (16) corrects a sign error in equation (4.9), and in the denominators of (4.6) and (4.8) of Newman (1978).

The inverse Fourier transforms of the last equations provide the relations

$$\sigma_j + U\hat{\sigma}_j + C_j(\sigma_j + \bar{\sigma}_j) = q_j, \tag{25}$$

$$2\pi i\, C_j\, \bar{\sigma}_j = \int_L q_j(\xi)\, f(x-\xi)\, d\xi, \tag{26}$$

where

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{-ikx}\, f^*(k). \tag{27}$$

After elimination of $C_j$ from (25) and (26), the outer source strength is determined from the integral equation

$$q_j(x) - \frac{1}{2\pi i}(\sigma_j/\bar{\sigma}_j + 1)\int_L q_j(\xi)\, f(x-\xi)\, d\xi$$

$$= \sigma_j(x) + U\hat{\sigma}_j(x). \tag{28}$$

Assuming a numerical solution for the two-dimensional potentials in (8), and the corresponding source-strengths $\sigma$, $\hat{\sigma}$, the integral equation (28) may be solved for the unknown outer source strength $q_j(x)$. The complete inner solution follows from (13) and (25), in the form

$$\varphi_j = \varphi_j^{(s)} + (2\pi i\bar{\sigma}_j)^{-1}(\phi_j+\bar{\phi}_j)\int_L q_j(\xi) f(x-\xi) d\xi. \tag{29}$$

The first term on the right side of (29) is the strip-theory potential (including the contribution from $\hat{\phi}_j$ which usually is ignored). The remaining contribution to (29) represents the three-dimensional interaction between adjacent sections.

In the high-frequency domain the integral in (29) tends to zero, and the strip-theory solution remains. Conversely in the low-frequency regime the two-dimensional potentials in (29) simplify and the ordinary slender-body result is recovered as derived by Newman and Tuck (1964). The unified potential (29) is valid more generally, for all wavenumbers between these two limiting regimes.

In the special case of zero forward velocity (U=0), the unified solution (29) reduces to a form closely related to the "interpolation solution" derived by Maruo (1970). Maruo's approach is rather different, but the only change in the final result is that the homogeneous solution $(\phi_j+\bar{\phi}_j)$ is replaced by (1+Kz), and the amplitude of the two-dimensional strip-theory potential is modified accordingly to satisfy the boundary condition on the body.

## 3. REDUCTION OF THE KERNEL

The kernel (27) in the integral equation (28) is defined by the inverse Fourier transform of the function $f^*$ given by (18) and (19). This kernel can be interpreted as the value of the source potential on the x-axis, after subtraction of the two-dimensional oscillatory source potential $G_{2D}$. Singularities can be expected, especially at x=0, and a careful analysis is required.

The singular behavior at x=0 can be mitigated by considering the integral of f(x), or the inverse transform of $f^*/(-ik)$. If this modification is offset by multiplying the transformed source strength $q_j^*$ by $(-ik)$, in (24), (29) is replaced by

$$\varphi_j = \varphi_j^{(s)} + (2\pi i\bar{\sigma}_j)^{-1}(\phi_j+\bar{\phi}_j)\int_L q_j'(\xi) F(x-\xi)\, d\xi. \tag{30}$$

Here $q_j'$ denotes the derivative of the source strength, and the new kernel is

$$F(x) = \frac{i}{2\pi}\int_{-\infty}^{\infty} f^*(k) e^{-ikx}\, dk/k. \tag{31}$$

Since $f^*=O(k)$ as $k\to 0$, the integral (31) is convergent and F(x) vanishes as $|x|\to\infty$. There is a logarithmic infinity in $f^*(k)$ as $|k|\to\infty$, and hence in F(x) as $x\to 0$, but this singularity can be integrated in (30) without difficulty.

The integral in (31) can be simplified by considering the function

$$\Lambda(k) = \ln(2K/k)$$
$$- (1-k^2/\kappa^2)^{-1/2}\, \ln[\kappa/k + (\kappa^2/k^2-1)^{1/2}], \tag{32}$$

where $\kappa(k)$ is defined by (16). $\Lambda(k)$ is analytic throughout the finite k-plane, except for a branch cut on the negative real axis. With appropriate values determined on each side of this branch cut, in the manner described by Sclavounos (1980), it follows that

$$f^*(k) = \Lambda(k\pm i0) + \pi i \pm \pi i\, H(-k)$$
$$+ (|1-k^2/\kappa^2|)^{-1/2}\, g_\pm(k)\,. \tag{33}$$

Here H(-k) is the Heaviside unit function, equal to one for k<0 and zero otherwise, and

$$g_+(k) = 2\pi i\,, \quad (-\infty<k<k_1), \tag{34a}$$

$$g_+(k) = 0\,, \quad (k_1<k<k_2), \tag{34b}$$

$$g_+(k) = -2\pi i\,, \quad (k_2<k<0), \tag{34c}$$

$$g_-(k) = 0\,, \quad (-\infty<k<0), \tag{34d}$$

$$g_\pm(k) = -\pi i\,, \quad (0<k<k_3,\ \tau<1/4), \tag{34e}$$

$$g_\pm(k) = \pi\,, \quad (k_3<k<k_4,\ \tau<1/4), \tag{34f}$$

$$g_\pm(k) = -\pi i\,, \quad (k_4<k<\infty,\ \tau<1/4), \tag{34g}$$

$$g_\pm(k) = -\pi i\,, \quad (0<k<\infty,\ \tau>1/4). \tag{34h}$$

The branch-points of the square-root function in (33) have been defined by

$$k_{1,2} = -(g/2U^2)[1+2\tau \pm (1+4\tau)^{1/2}], \tag{35}$$

$$k_{3,4} = (g/2U^2)\ [1-2\tau \mp (1-4\tau)^{1/2}], \tag{36}$$

and

$$\tau = \omega U/g. \tag{37}$$

Note that $k_{1,2}$ are real and negative, whereas $k_{3,4}$ are positive for $\tau<1/4$, and complex- conjugate otherwise.

From Jordan's lemma

$$\int_{-\infty}^{\infty} \Lambda(k\pm i0)\ e^{-ikx}\ dk/k = 0, \quad (x \lessgtr 0). \tag{38}$$

Hence, from (31) and (33),

$$F(x) = F_1(x) + F_2(x), \quad (x<0), \tag{39a}$$

$$F(x) = F_2(x), \quad (x>0), \tag{39b}$$

where

$$F_1(x) = -\int_{-\infty}^{k_1} e^{-ikx}[1+(1-k^2/\kappa^2)^{-1/2}]dk/k$$

$$-\int_{k_1}^{k_2} e^{-ikx}\ dk/k$$

$$-\int_{k_2}^{0} e^{-ikx}[1-(1-k^2/\kappa^2)^{-1/2}]dk/k, \tag{40}$$

and

$$F_2(x) = -\frac{1}{2}\left(\int_0^{k_3}+\int_{k_4}^{\infty}\right)e^{-ikx}[1-(1-k^2/\kappa^2)^{-1/2}]dk/k$$

$$-\frac{1}{2}\int_{k_3}^{k_4} e^{-ikx}[1-i(k^2/\kappa^2-1)^{-1/2}]dk/k, \quad (\tau<1/4), \tag{41a}$$

$$F_2(x) = -\frac{1}{2}\int_0^{\infty} e^{-ikx}[1-(1-k^2/\kappa^2)^{-1/2}]dk/k, \quad (\tau>1/4). \tag{41b}$$

The function $F_1$ is logarithmically infinite at $x=0$, but $F_2$ is regular at this point for $U>0$. From (39) it follows that the logarithmic singularity exists only on the downstream side of $x=0$.

The singularity in $F_1$ can be displayed explicitly by using properties of the sine and cosine integrals to express (40) in the alternative form

$$F_1(x) = \left(\int_{-\infty}^{k_1} - \int_{k_2}^{0}\right)e^{-ikx}[1-(1-k^2/\kappa^2)^{-1/2}]dk/k$$

$$-\left(\int_{k_1}^{0} + \int_{k_2}^{0}\right)(1-e^{-ikx})dk/k$$

$$-\ 2[\ell n(\omega|x|/U) + \gamma + \pi i/2]. \tag{42}$$

Here $\gamma=0.577...$ is Euler's constant. The integrals in (41) and (42) are convergent for all values of x, if $U>0$, and can be evaluated by numerical quadratures.

Both limiting forms of the kernel, for zero forward velocity and for zero frequency, can be derived by letting $\tau\to 0$. The resulting integrals in (40) and (41a) are evaluated after replacing the branch-points (35) and (36) by their limiting values, $k_{1,4} \cong \mp(g/U^2)$ and $k_{2,3} \cong \mp K$, and approximating $\kappa$ by $U^2k^2/g$ or $K$, respectively. In this manner it can be shown that, for $\tau=0$,

$$F(x) = \pm \frac{1}{2}[\ell n(2K|x|) + \gamma + \pi i]$$

$$\mp \frac{\pi}{4} \int_0^{K|x|} [\mathbf{H}_o(t)+Y_o(t)+2i\, J_o(t)]dt$$

$$- \frac{\pi}{4}[(2\mp 1)\; Y_o(g|x|/U^2) - \mathbf{H}_o\,(gx/U^2)],$$

$$(x \gtrless 0). \qquad (43)$$

Here $\mathbf{H}_o$, $Y_o$ and $J_o$ are the Struve and Bessel functions of order zero.

The contribution from the last line in (43) vanishes for U=0, and the resulting kernel is equivalent to that derived by Ursell (1962). In this case, as in classical slender-body theory without a free surface, the logarithmic singularity is antisymmetrical.

For the steady-state case $\omega=0$, on the other hand, the integral in (43) vanishes and the result is consistent with that of Tuck (1963). As $x\rightarrow+0$, the resulting singularity from the first term on the right-hand side of (43) is cancelled by the Bessel function $Y_o$ and, as stated above for the more general unsteady case, there is no upstream logarithmic singularity. (The contribution from $\ell n(K)$ in the first term is cancelled by a similar factor in the low-frequency limit of the two-dimensional source potential $G_{2D}$.)

The regular part of the kernel (27) as a function of x/L is shown in Figure 1 for a Froude number 0.2 and two values of $\tau$, 0.2 and 0.7 less and greater than 1/4 respectively.

## 4. ADDED-MASS AND DAMPING COEFFICIENTS

The principal application of the results above is to predict the hydrodynamic pressure force and moment, acting upon a heaving and pitching ship hull in response to its oscillations. With the usual decomposition, these forces and moments are expressed in terms of added-mass ($a_{ij}$) and damping ($b_{ij}$) coefficients, which are the factors of the acceleration and velocity, respectively, in a linear expression for the total force and moment. Here i=3, for the heave force, i=5 for the pitch moment, and j=3,5 respectively for the contribution due to each mode.

A total of eight coefficients must be considered, including cross-coupling between heave and pitch. These coefficients can be derived from the inner velocity potential (29) or (30), by means of Bernoulli's equation for the linearized pressure, and after using a theorem due to E. O. Tuck (Ogilvie and Tuck, 1969) the results can be summarized in the form

$$\omega^2 a_{ij} - i\omega b_{ij} = -i\omega\rho \iint n_i \phi_j \, dS$$

$$- \rho\, U \iint (i\omega n_i \hat{\phi}_j - m_i \phi_j) dS + \rho U^2 \iint m_i \hat{\phi}_j \, dS$$

$$- \rho \iint C_j(x)(i\omega n_i - U m_i)\; (\phi_j + \bar{\phi}_j)\; dS. \qquad (44)$$

Here the surface integrals are over the submerged portion of the hull and, except for

$$m_5 = n_3 - x m_3, \qquad (45)$$

the quantities in (44) are defined in Section 2.

The first integral in (44) is the zero-speed strip-theory contribution, or the integral along the length of the two-dimensional added-mass and damping coefficients. The second and third integrals in (44) represent linear and quadratic effects of the forward velocity which appear (to varying degrees) in the strip theories. (The quadratic terms are sometimes regarded as higher-order, and the potential $\hat{\phi}$ is usually ignored.) Green's theorem can be used to show that the second integral in (44) vanishes when i=j.

The last integral in (44) represents the three-dimensional correction from the interaction function $C_j(x)$. As $\omega\rightarrow\infty$, the integral equation (29) can be used to show that $C_j\rightarrow 0$, and the "pure" strip theory is recovered. Except for this limiting case, however, three-dimensional effects are significant in (44).

The first computations of added mass and damping based on the unified theory were performed by Mays (1978) for a prolate spheroid, floating with its major axis in the plane of the free surface, and for zero forward velocity (U=0). From symmetry considerations there is no cross-coupling in this case. Comparisons with the ordinary slender-body theory, strip theory, and with "exact" three-dimensional numerical solutions are included by Mays for values of the beam-length ratio equal to 1/16, 1/8, and 1/4. The results for 1/8 are reproduced in Figure 2 and it is apparent that the added-mass and damping coefficients predicted by the unified slender-body theory are in virtually perfect agreement with the exact solutions of Kim (1964) and Yeung (Bai and Yeung, 1974). By comparison, the strip theory predictions are satisfactory only for relatively high frequencies (KB>1), and the ordinary slender-body theory is useful only for KL<1. For the beam-length ratio equal to 1/4 May's computations show almost the same degree of agreement, and demonstrate the broad range of applicability of the unified theory for zero forward velocity.

Our first computations with nonzero forward velocity were performed for a floating spheroid of beam-length ratio 1/6, for comparison with the experiments of Lee and Paulling (1966). The results were generally in agreement, but the experimental scatter precludes a definitive judgement of the degree of improvement of the unified theory relative to strip theory.

Subsequent computations were performed for two realistic hull forms where experimental data is available. In each case we show the computations based on unified theory, and the strip theory results of Salvesen, Tuck and Faltinsen (1970).

The results for a Series 60 hull (block coefficient 0.7) are shown in Figures 3 and 4, and compared with the experimental data of Gerritsma and Beukelman (1964) and Gerritsma (1966). For zero forward velocity (Figure 3) the agreement between the unified theory and experiments is very good for $a_{33}$, $b_{33}$, and $b_{55}$. The remaining coefficients show a departure of the experimental data at low frequencies. The cross-coupling coefficients are symmetric in this case, and only one pair are shown. For low and moderate frequencies the differences between the unified and strip theories are substantial, and the experiments generally support the unified theory. All eight coefficients are shown in Figure 4, for a Froude number of 0.2. In this case the differences between the two theories are reduced, suggesting in the strip theory that there is some cancellation between the approximations associated with forward velocity and three-dimensionality. The agreement between the unified theory and experiments is generally favorable, with the notable exception of the cross-coupling coefficients $a_{35}$, $a_{53}$, and $b_{53}$.

The coefficients $a_{33}$, $a_{55}$, $b_{55}$ and $a_{35}$ have also been computed for the Series 60 hull by Chang (1977), using a full three-dimensional theory but neglecting the contribution from the potential $\hat{\phi}$. For zero forward velocity Chang's results are indistinguishable from the unified theory. For the Froude number 0.2, the same is true of $a_{33}$ and $a_{55}$, whereas Chang's comparison with experiments is better for $a_{53}$ and worse for $b_{55}$.

Our final results are for the Friesland class frigate hull (block coefficient 0.554) where experimental data are given by Smith (1966). The comparisons in Figures 5 and 6 are for Froude numbers of 0.15 and 0.35, respectively. Once again there is a tendency in some coefficients for the experimental data to diverge from the unified theory at low frequencies, and the cross-coupling coefficient $a_{35}$ shows poor comparison for all frequencies. The remaining results for the lower Froude number show good to excellent agreement between the unified theory and experiments. Similar conclusions apply for the higher Froude number, except that in this case the comparison for the coefficient $b_{53}$ is unsatisfactory. In this case, unlike the Series 60 hull, there is good agreement for the coefficient $a_{53}$.

These comparisons of the added-mass and damping coefficients can be summarized with the following conclusions. In the case of zero forward velocity excellent agreement exists between the unified theory, three-dimensional numerical solutions, and experimental data. With forward velocity included, there are no complete three-dimensional computations with which to compare, and the unified theory can be judged only on the basis of experiments. Good agreement exists in most cases, but the confirmation is not satisfactory for some of the cross-coupling coefficients. Relative to the strip-theory predictions with forward speed, the unified theory provides a noticable improvement in the diagonal coefficients $a_{33}$, $a_{55}$, $b_{33}$ and $b_{55}$.

Although the accuracy of the experimental data is not well established, one possible explanation for the remaining discrepancies is that the treatment of end effects in the unified theory requires some refinement. In this context we note that the steady-state disturbance potential $U\overline{\phi}$ is approximated in the inner region in a stripwise manner by assuming no interaction between subsequent cross-sections. A wall boundary condition is satisfied on the free surface and conformal mapping is used for the evaluation of $m_3$ through expression (5). The two dimensional velocity potentials $\hat{\phi}_j$ satisfying a wave free-surface condition are then evaluated using a two dimensional numerical procedure due to Yeung (1975).

This procedure breaks down at the ship ends, introducing a significant overprediction of $m_3$ and consequently of $\hat{\sigma}_3$ and $\hat{\sigma}_5$. This difficulty has been avoided by assuming a linear variation of $m_3$ within 5% of the ship length away from each end, and assuming $m_3=0$ at the ends. This problem could be overcome by evaluating $m_3$ from the full three-dimensional double-body steady disturbance potential.

The kernel of the integral equation defined in (41) and (42) was evaluated numerically using Simpson's integration formula, with appropriate truncation corrections based on asymptotic expansions of the integrand. The number of integration points is determined to ensure a relative error less than $10^{-5}$.

The integral equation (28) is solved by iteration using the strip-theory source distribution as the first iteration. The solution obtained in this manner has been checked against an independent matrix-inversion solution.

The two-dimensional strip-theory calculations were performed on an IBM370. The kernel evaluation and the solution of the integral equation were performed on a PDP11-34 minicomputer. The computation times required are estimated as follows:

TABLE 1

COMPUTATION TIMES

| | IBM370 (sec) | PDP11-34 (sec) |
|---|---|---|
| 2D potentials ($\phi_3$) | 3 | 180 |
| 2D potentials ($\hat{\phi}_3$) | 4 | 240 |
| Kernel and integral equation (U=0) | 0.16 | 10 |
| (U>0) | 1 | 60 |
| Total time required (U=0) | 3.16 | 190 |
| (U>0) | 8 | 480 |

These estimates suggest that for finite forward velocity the additional computational effort required by unified theory is of the order of 1/7 of the two-dimensional strip-theory calculations if the latter are complete. For zero forward velocity the corresponding ratio is 1/18.

## 5. ELONGATED WAVE-POWER DEVICES

The unified theory can be used to analyse the performance of elongated wave-power absorbers such as the Cockerell raft and Kaimei ship, in a similar manner to the results of Newman (1979) based on the ordinary slender-body theory. In this application the forward velocity is set equal to zero, with resulting simplification of the analysis.

Following Newman (1979), we consider the power absorbed by a slender body moored in the head-sea configuration and performing vertical oscillatory motions of appropriate amplitude and phase, along its length. The power absorbed by this motion can be represented as the product of the energy flux per unit width in the incident wave system, and an "absorption width" W. In ideal circumstances W is comparable to the wavelength or body length, and substantially larger than the projected width of the body.

The absorption width can be expressed in terms of the far-field radiated wave amplitude due to the body motions, or the Kochin function $H(\theta)$ which is proportional to the radiated wave amplitude in the direction $\theta$ relative to the x-axis. If the incident waves propagate in the +x-direction, and if the body motions are controlled in an optimum manner to maximize the absorption width, this quantity can be expressed in the form

$$W = \left(\frac{2\pi}{K}\right)\frac{|H(\pi)|^2}{\int_0^{2\pi}|H(\theta)|^2\,d\theta} \tag{46}$$

In long wavelengths the optimum modal amplitudes of the body increase in proportion to the wavelength, and unrealistically large motions are required for (46) to be valid. To estimate the practical limit of the absorption width we define a parameter $\beta$ as the product of the beam-length ratio (b/L) and the maximum allowed vertical displacement per unit wave amplitude. Assuming arbitrarily that the maximum displacement is twice the incident wave amplitude, and that the beam-length ratio is between 0.1 and 0.2, typical values for $\beta$ are 0.2 and 0.4, respectively.

With the body motions limited in the above sense, the absorption width is given by

$$W = 2\beta|LH(\pi)/b| - \frac{1}{2\pi}\beta^2 K\int_0^{2\pi}|LH(\theta)/b|^2\,d\theta \tag{47}$$

for small values of $\beta$, and by (46) when $\beta$ is larger than the value where (47) attains its maximum. Alternatively, with $\beta$ fixed, (46) holds for $K>K_o$ and (47) for $K<K_o$, where the transition wavenumber is defined by the condition that (46) and (47) are equal.

In ordinary slender-body theory, where $Kb<<1$, the Kochin function is given to leading order by

$$H(\theta) = -K\int_L f(\xi)\,b(\xi)\,e^{-ik\xi\cos\theta}\,d\xi. \tag{48}$$

Here f(x) is the vertical displacement and b(x) is the local beam at the waterplane. This approximation was used by Newman (1979) to calculate the absorption width of various modal shapes, with the symmetric and anti-symmetric modes (with respect to x) treated separately and superposed to obtain the total absorption width. The curves in Figure 7 show the total absorption width for an articulated raft, consisting of three rigid segments connected by two symmetric hinges.*

In the unified theory, the Kochin function can be expressed in terms of the outer source strength q(x), and (48) is replaced by

$$H(\theta) = -\frac{i\omega}{2g}\int_L q(\xi)\,e^{-ik\xi\cos\theta}\,d\xi, \tag{49}$$

*For this case, and also for the Legendre polynomial modes, the values of $\beta$ given by Newman (1979) should be multiplied by a factor of 2.0. This error has been corrected in Figure 7.

with q(x) determined from the integral equation (28). Computations have been performed on this basis, for an articulated raft with beam-length ratio 0.1 and beam-draft ratio 2.0. These new results are shown in Figure 7, and a comparison can be made with the absorption width based on the ordinary slender-body theory. This comparison reveals that the latter approximation overestimates the absorption width by a substantial amount, when the modal amplitudes are limited, but in the shorter wavelength regime where this limitation is not applicable, the ordinary slender-body theory is quite accurate. Similar conclusions have been reached by Haren (1980) based on a three-dimensional numerical solution in the case of a body with zero draft. It appears that the ordinary slender-body theory overpredicts the magnitude of the Kochin function, and hence the limited absorption width (47), but (46) is not sensitive to this error in view of its form.

In conclusion, the earlier results of Newman (1979) based on the use of ordinary slender-body theory overpredict the performance of an elongated wave-power device, particularly in the regime of wavelengths where the absorption width is a maximum. The unified theory can be used to provide a more precise estimate of the absorption width.

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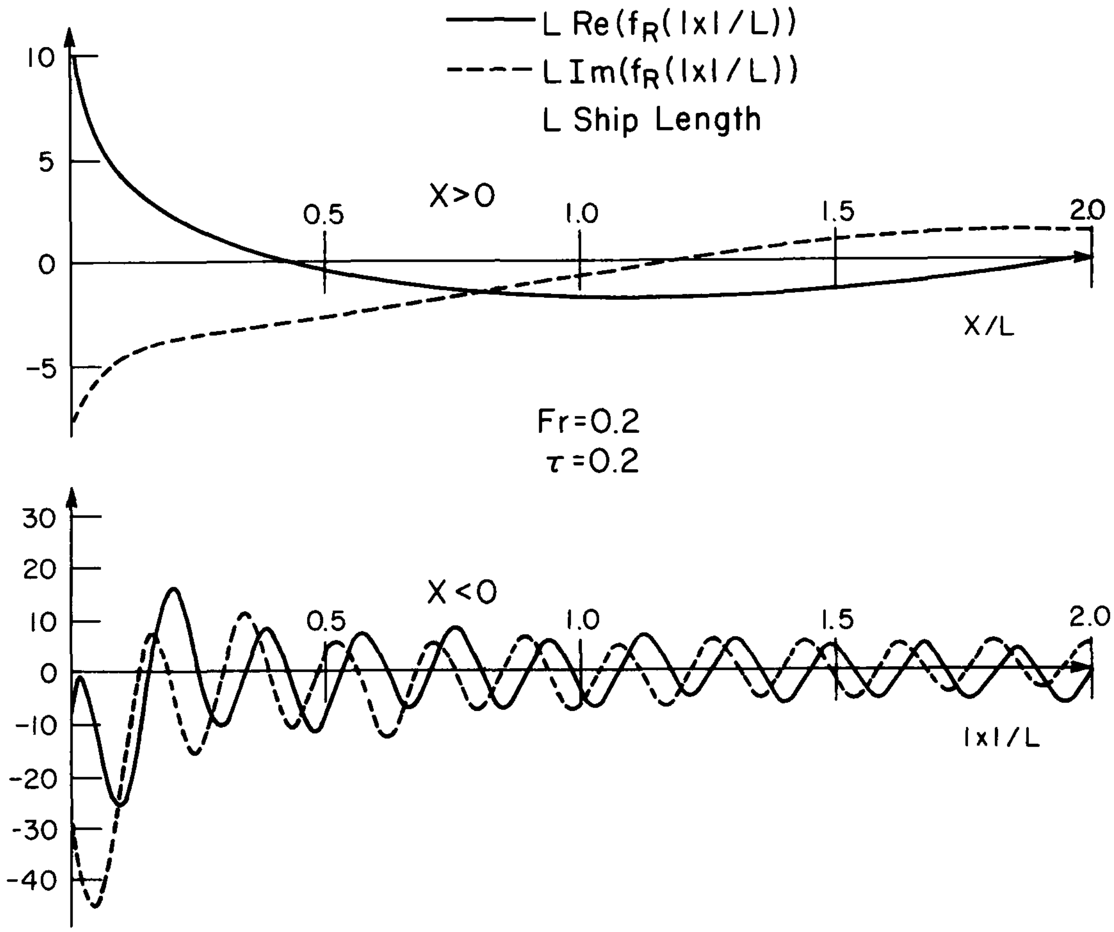


Figure 1a - Regular part ($f_R$) of the kernel (27) as a function of the longitudinal coordinate $|x|/L$ for $\tau=\omega U/g$ equal to 0.2 and 0.7 (Figures 1a and 1b respectively). Waves are present upstream only for the first case, associated with the root $k_3$ in (36) and with the wavelength-to-ship-length ratio $2\pi/k_3L \doteq 3.3$. For $\tau=0.7$ (Figure 1b) no waves exist upstream. Downstream of the disturbance the most obvious wave motion is associated with the largest root $k_1$, and with the wavelength-to-ship-length ratio 0.19 ($\tau=0.2$) and 0.12 ($\tau=0.7$). Longer wavelengths also exist downstream, associated with the roots $k_2$ and $k_4$ for $\tau < \frac{1}{4}$ and with $k_2$ alone for $\tau > \frac{1}{4}$. Their superposition upon the shorter wave system is more apparent in Figure 1b.

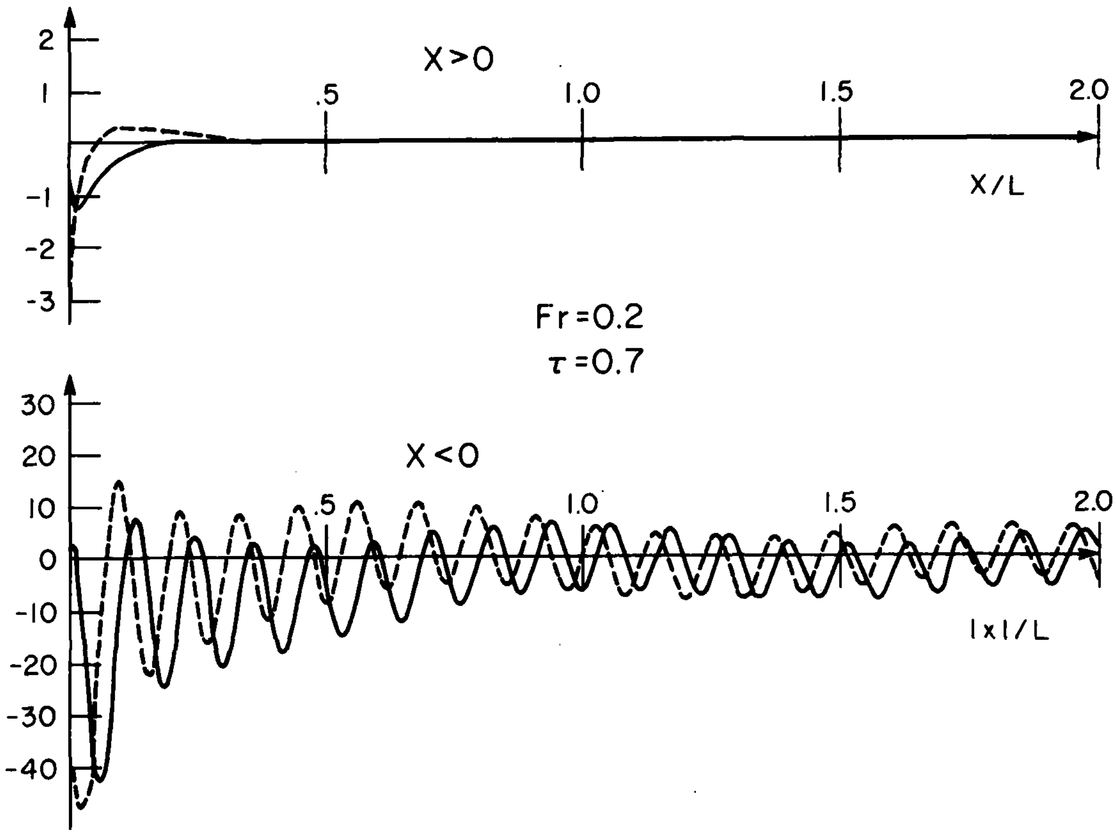


Figure 1b - see Figure 1a

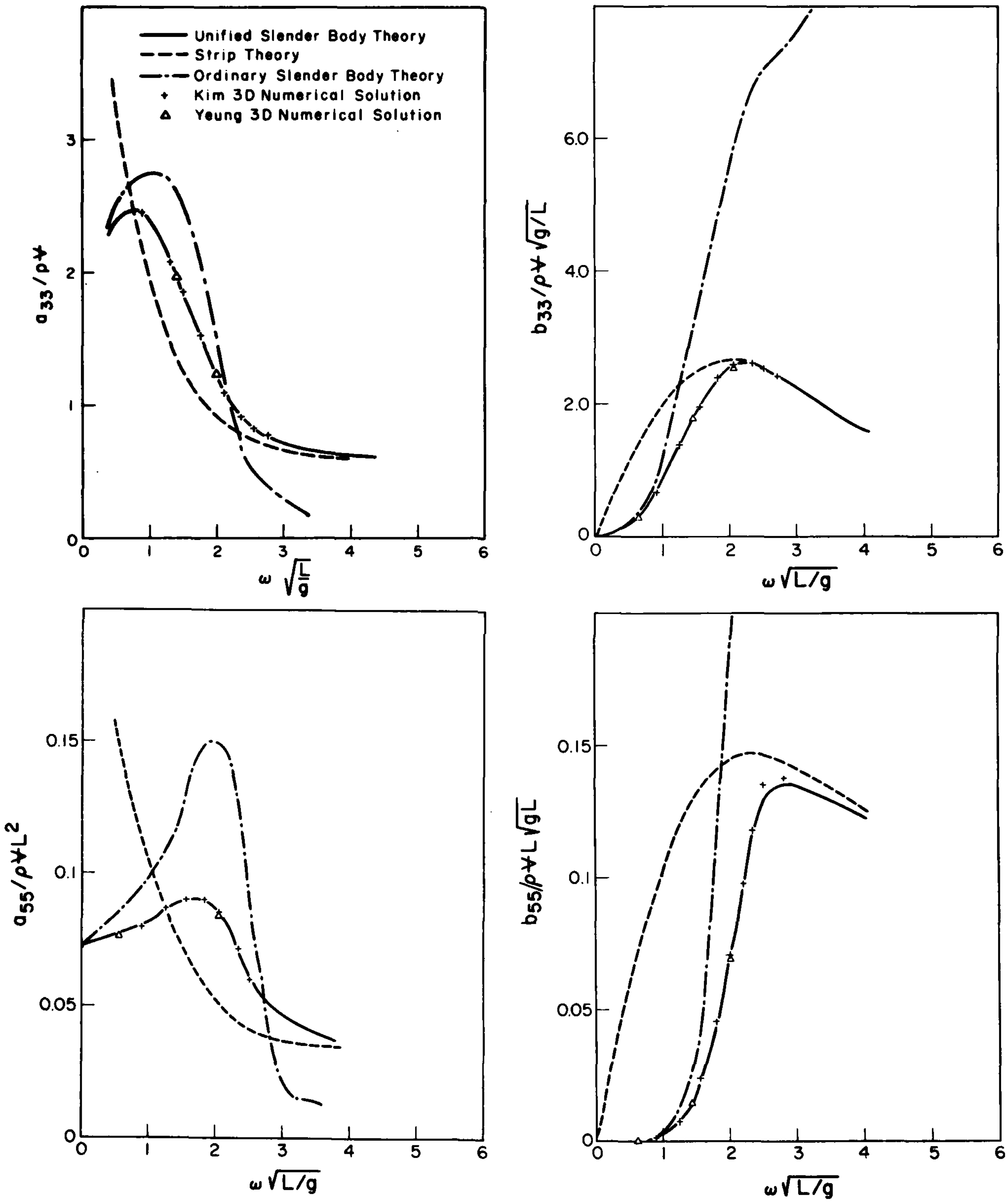


Figure 2 - Added-mass and damping coefficients of a prolate spheroid (b/L = 1/8) at U=0.

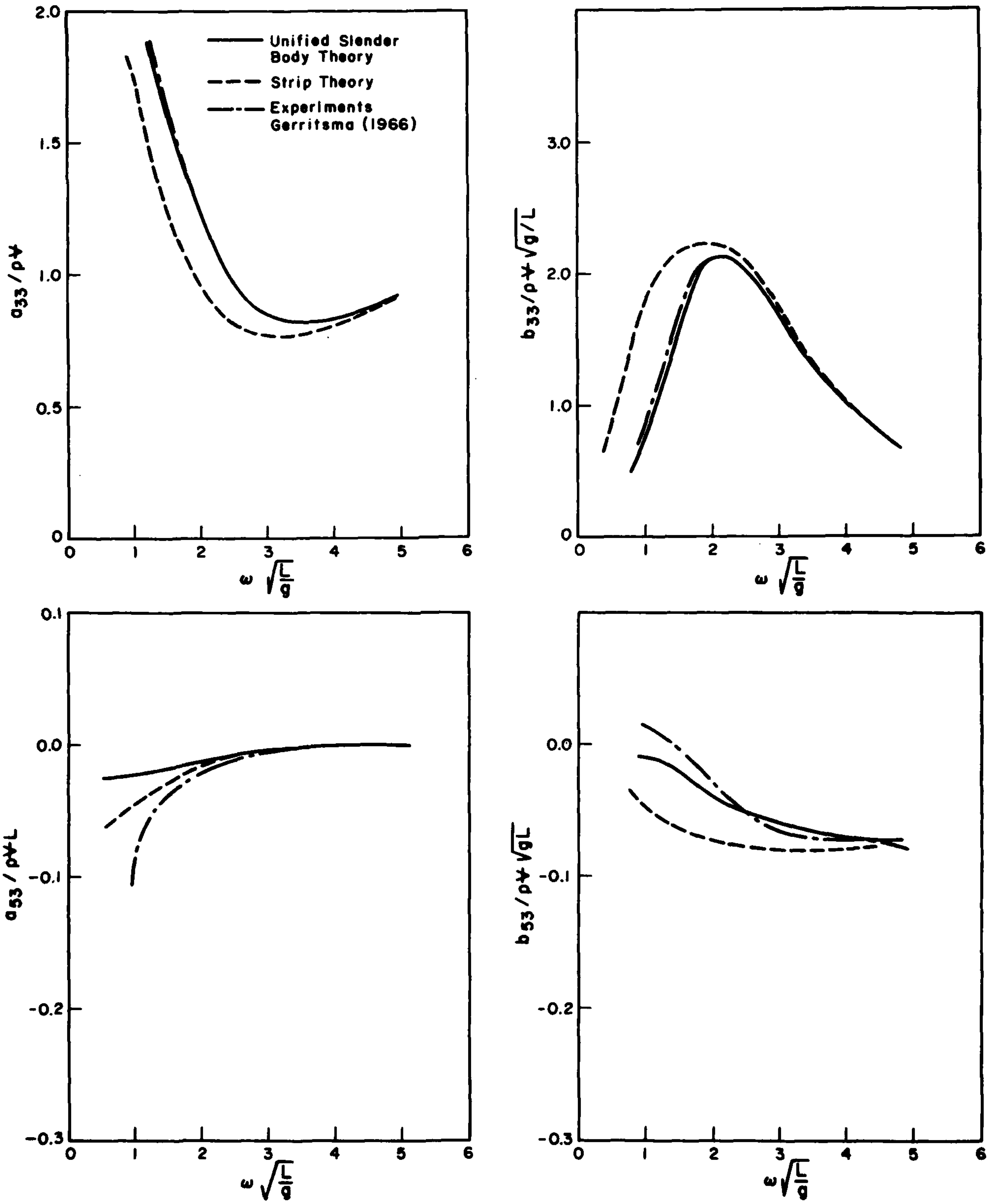


Figure 3 - Added-mass and damping coefficients of a Series 60 hull ($C_B$ = 0.7) at U=0.

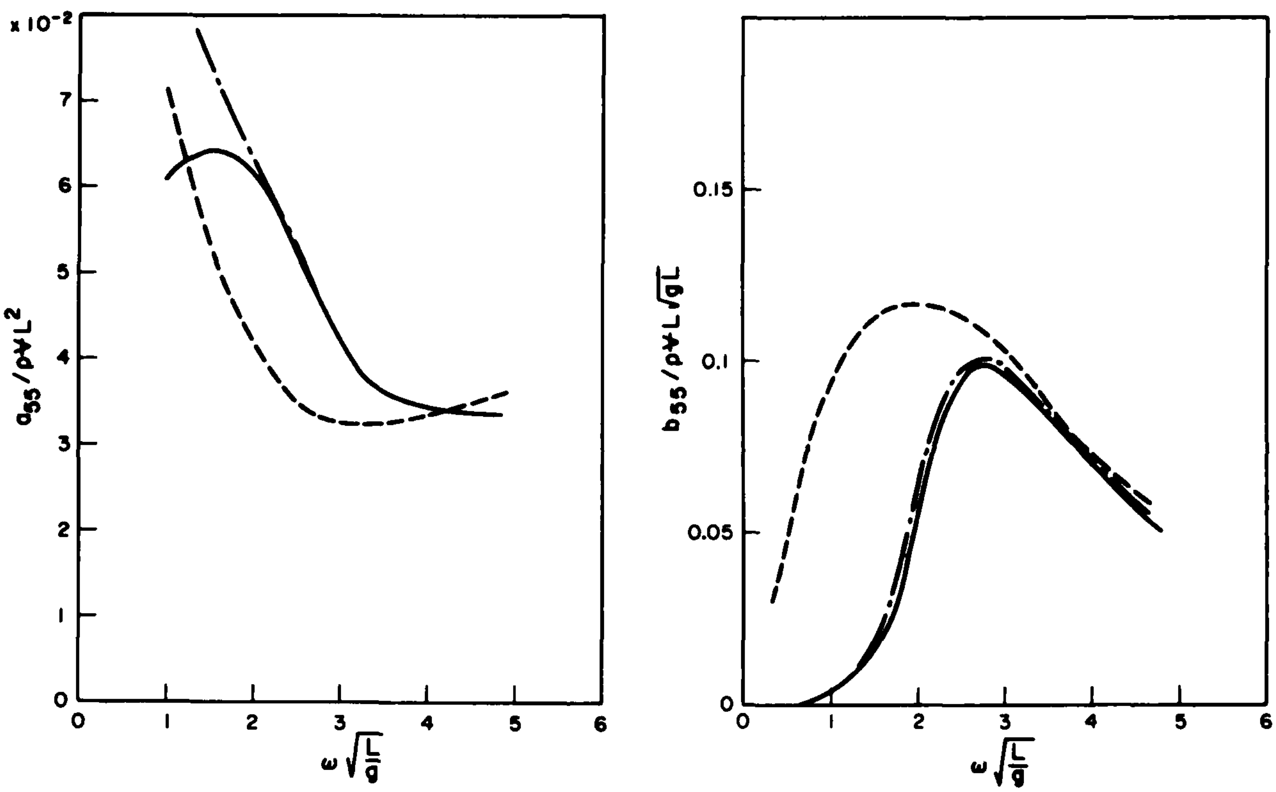


Figure 3 - continued

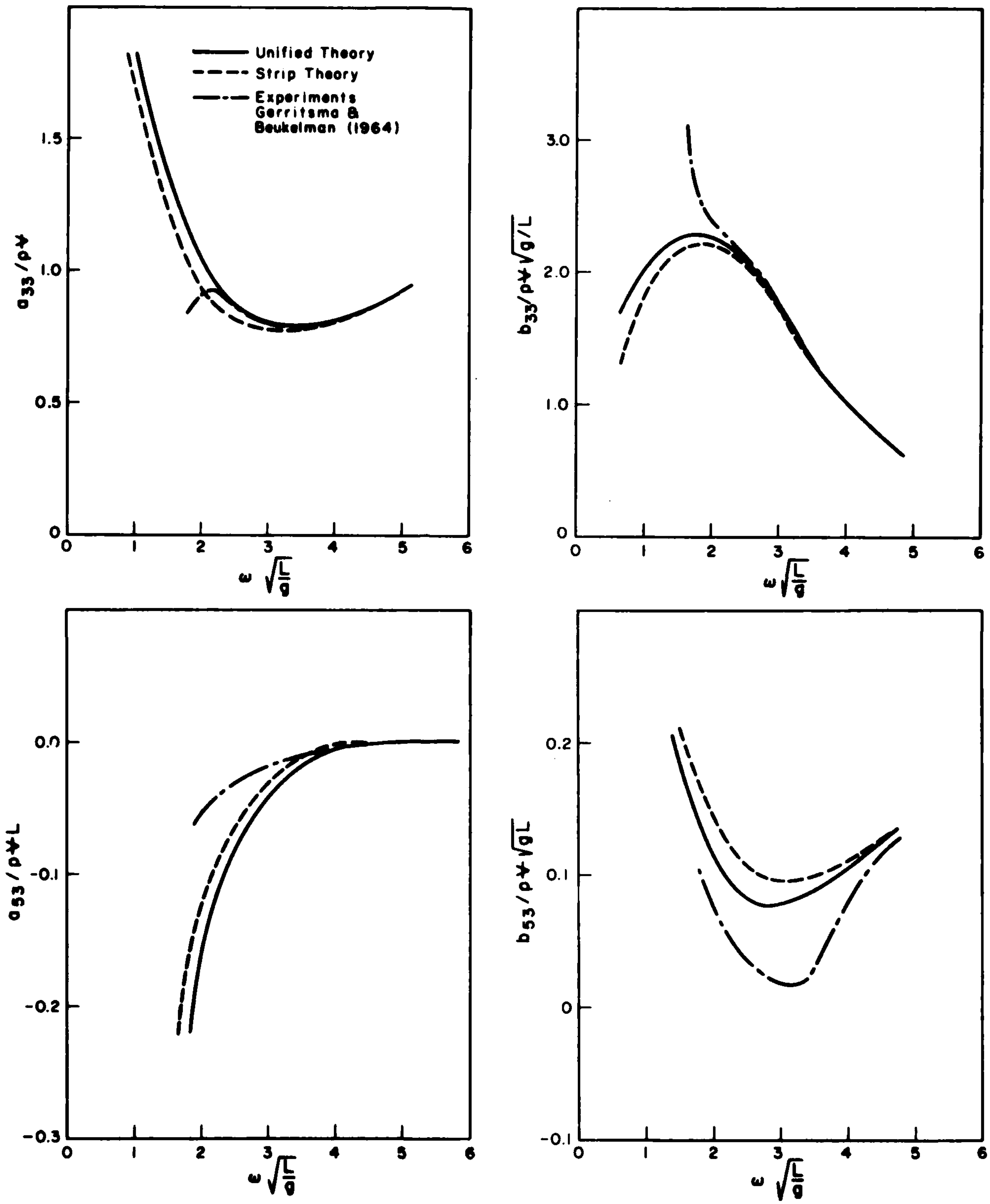


Figure 4 - Added-mass and damping coefficients of a Series 60 hull ($C_B$ = 0.7) at Fr=0.2

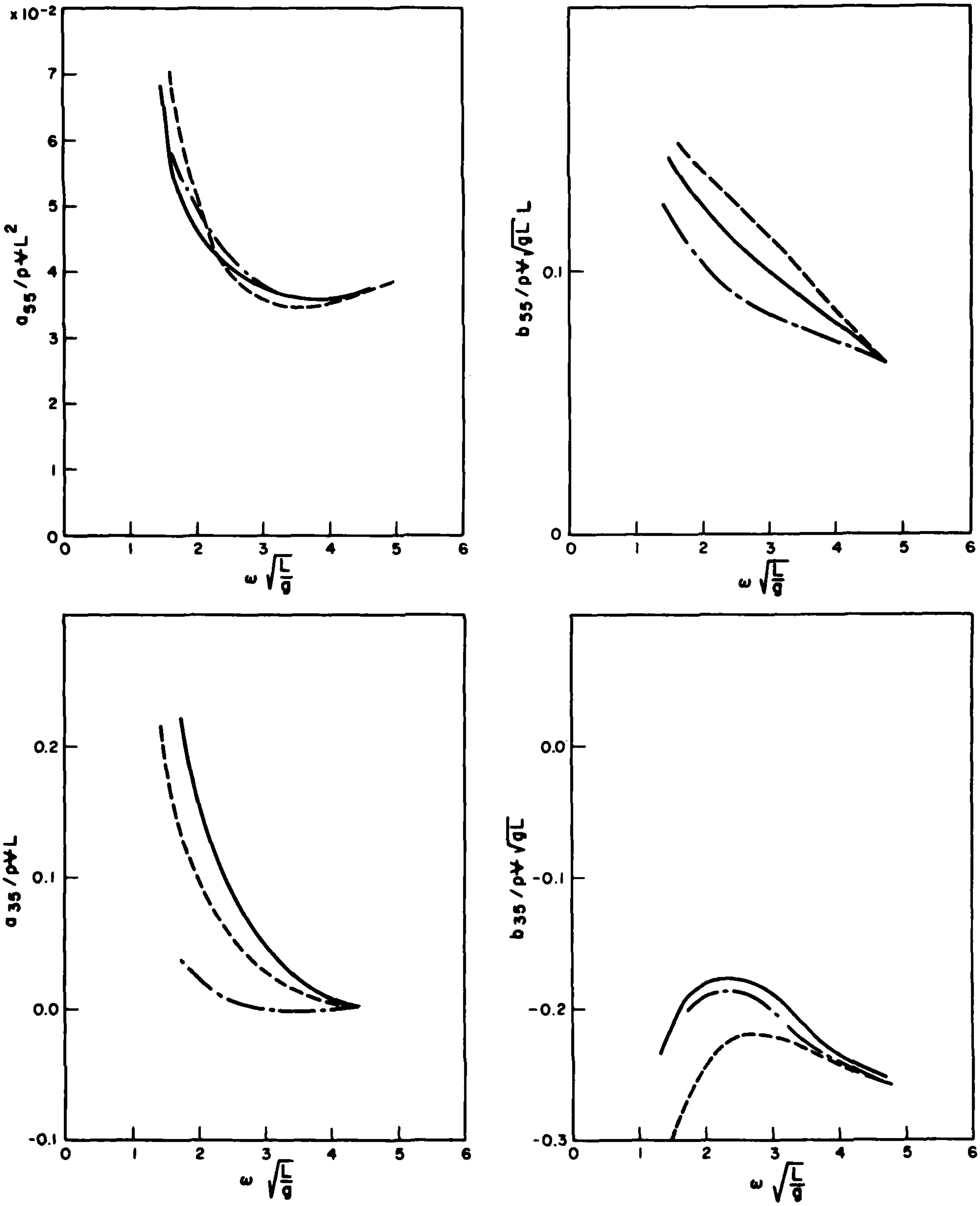


Figure 4 - continued

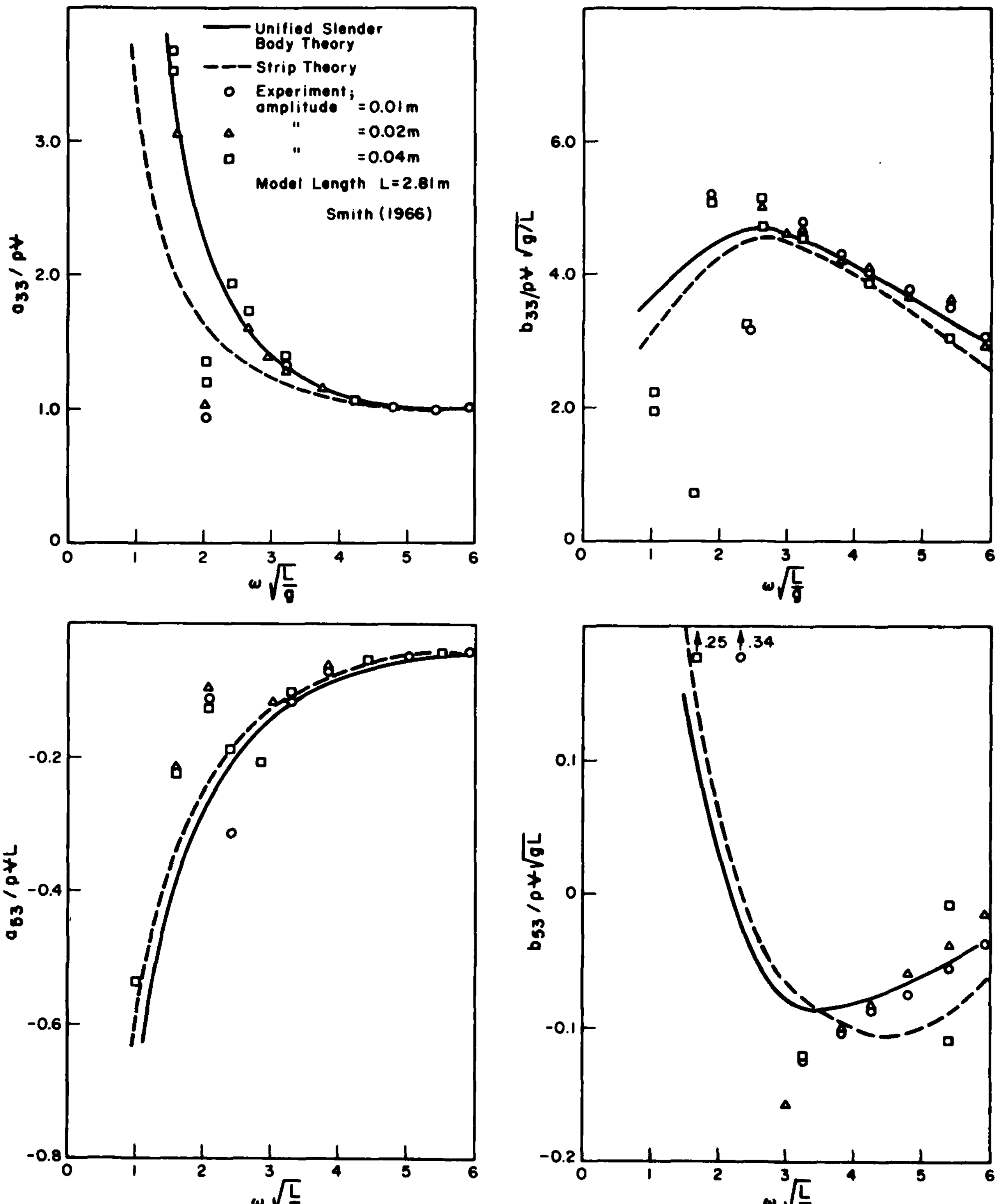


Figure 5 - Added-mass and damping coefficients of a frigate hull ($C_B$ = 0.55) at Fr=0.15.

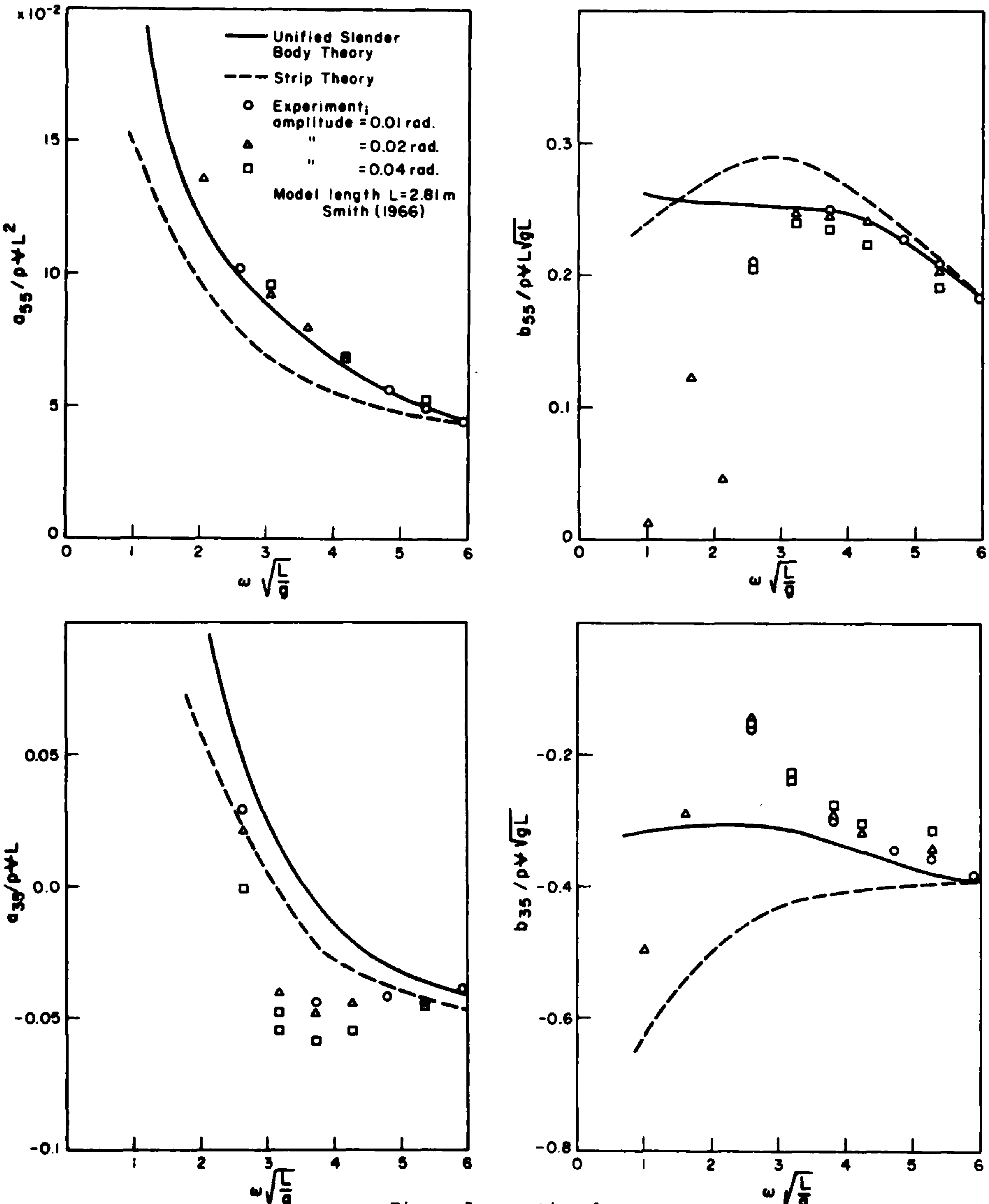


Figure 5 - continued

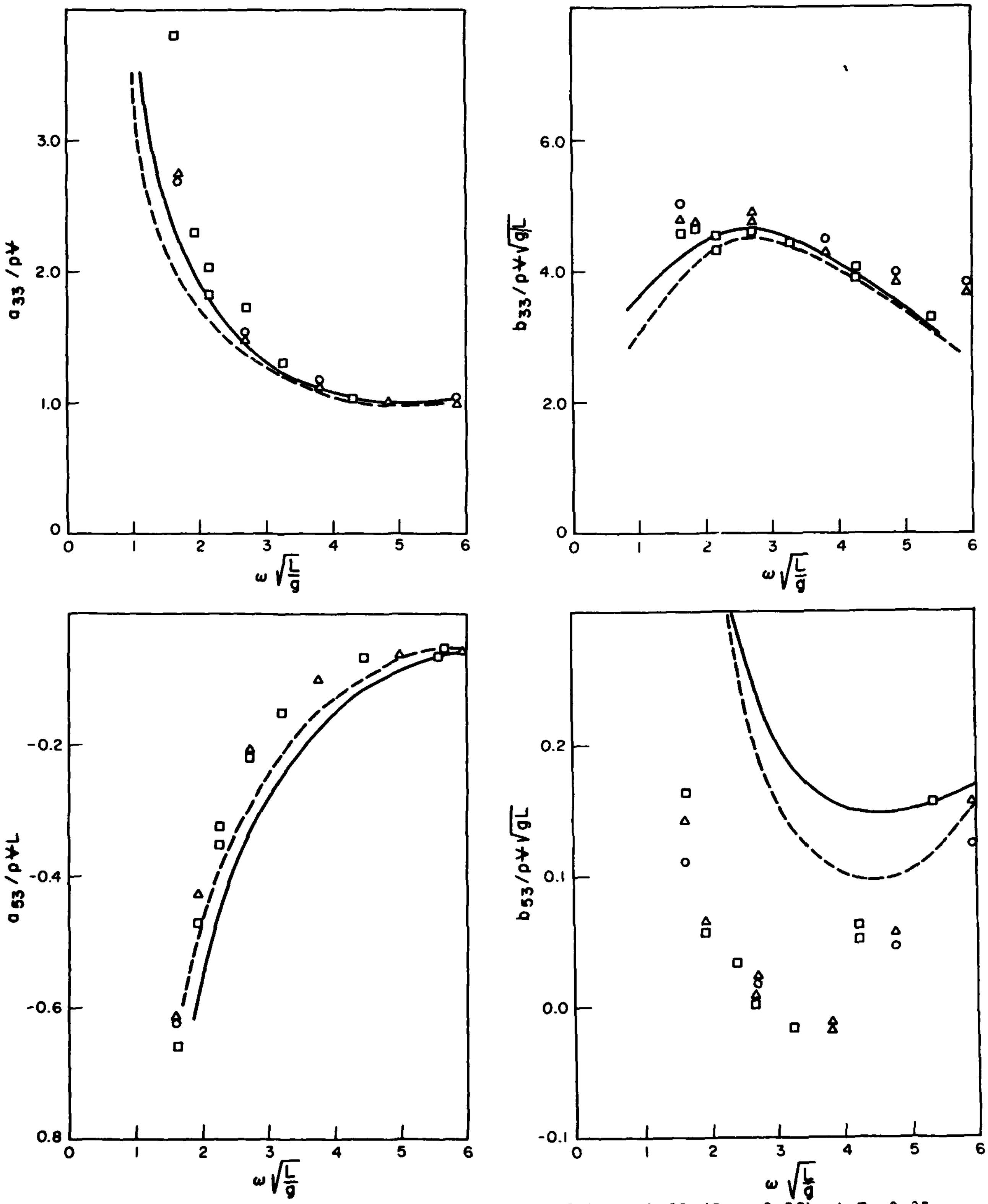


Figure 6 - Added-mass and damping coefficients of a frigate hull ($C_B$ = 0.55) at Fr=0.35.

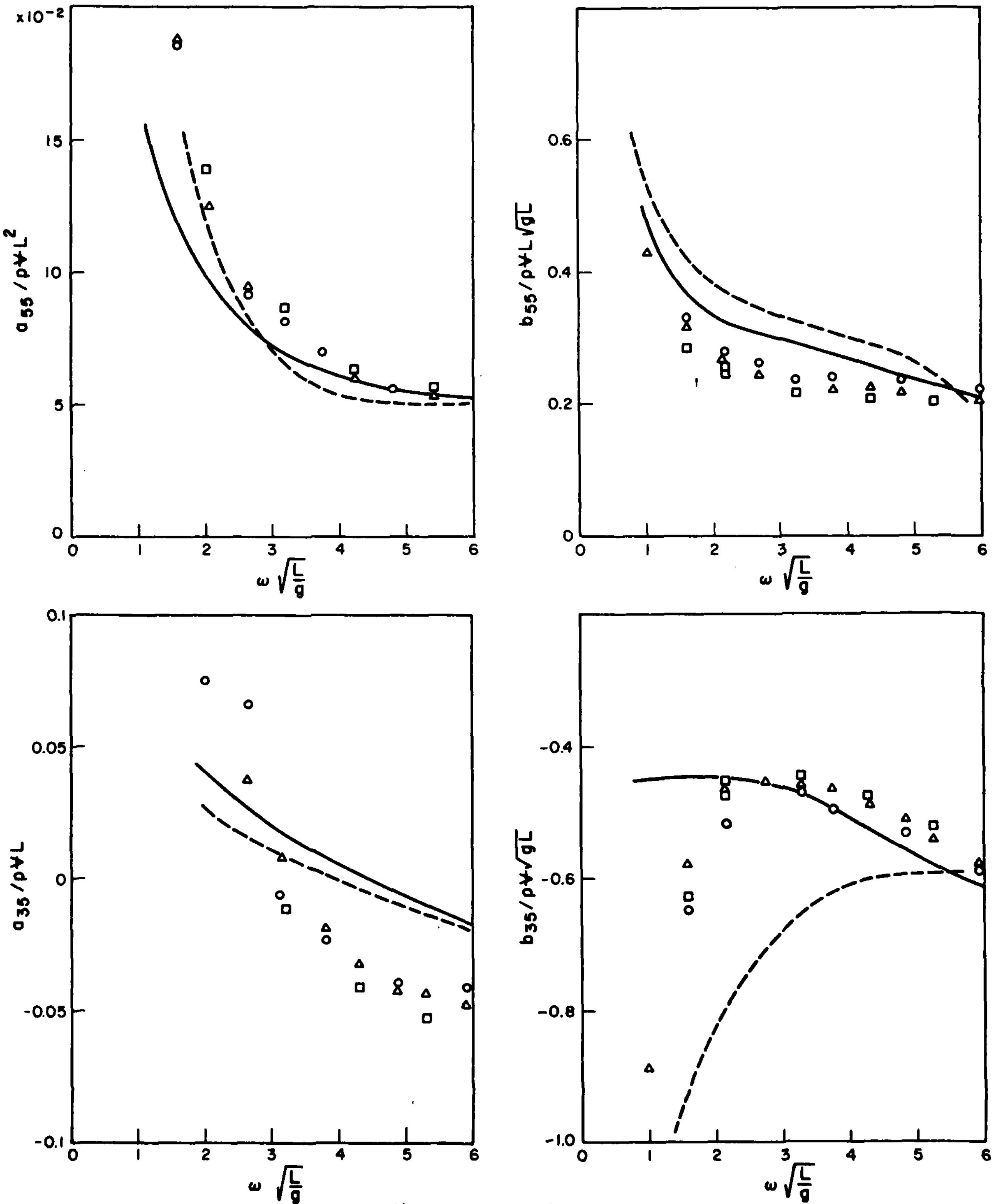


Figure 6 - continued

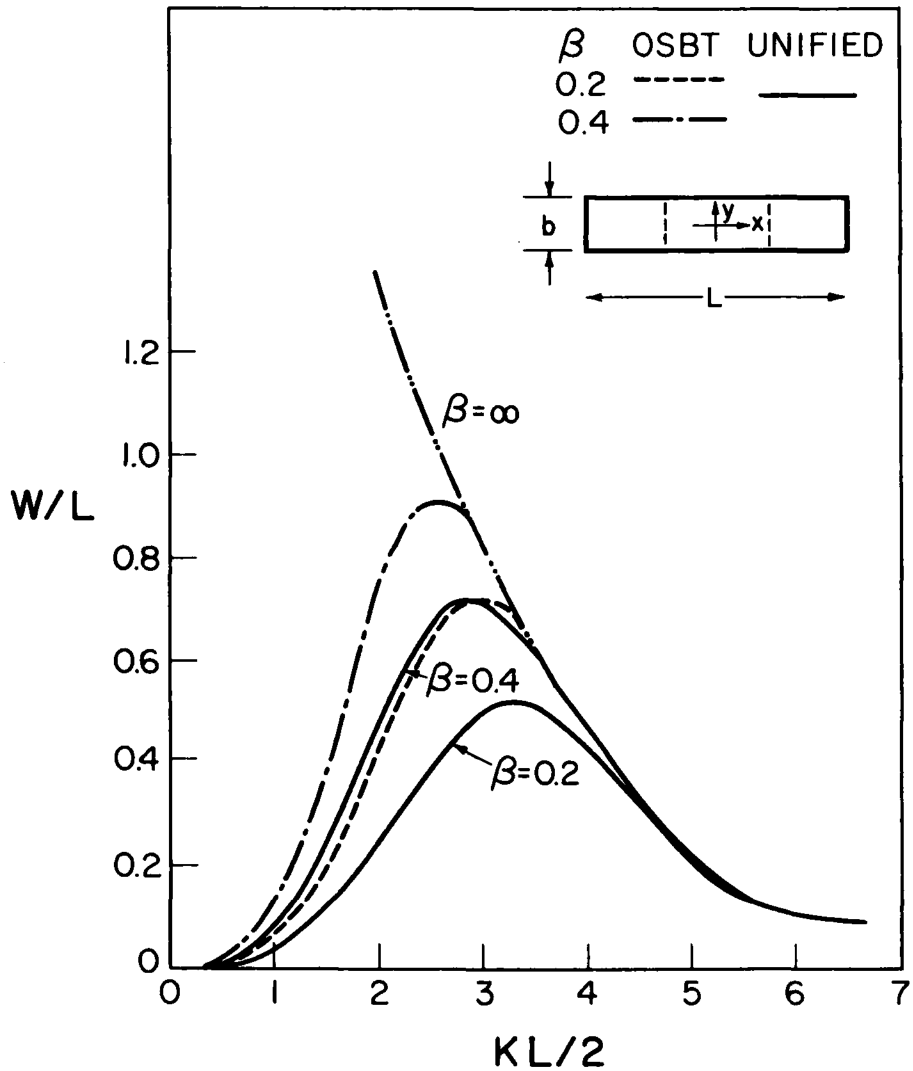


Figure 7 - Total absorption width, as a fraction of the body length, for an articulated raft with two symmetric hinges situated at the points $x = \pm L/5$. The curves for the ordinary slender-body theory (OSBT) are from Newman (1979), corrected as noted in the footnote, with superposition of the power obtained separately in the even and odd modes. The full lines are the values of the absorption width determined from the unified theory, for a beam-length ratio of 0.1, and a beam-draft ratio of 2.0. In all cases the body motions are of optimum phase. The maximum amplitude in each mode is equal to the product of $\beta L/b$ and the incident wave amplitude.